

Teaching Mathematics through Progressions

Definition:

Mathematics is:

The study of things that can be

counted and ***measured***

and the ***relationships*** between
them.

A **progression** is a method (*an organized table*) to allow students to access "higher" levels of mathematics by examining the relationship of quantities that can be counted and measured, and how those relationships change.

Students can practice their **arithmetic** and **algebraic** "skills" while developing progressions.

Through progressions, students can examine: ***ratios, proportions, percents, equations, and expressions***. The acquisition of graphing skills and understanding of functions is strongly linked to the conceptual understanding of these topics

These topics comprise approximately 80% of the middle school curriculum and are central to the acquisition and understanding of algebra.

Quantities (or attributes) of objects that can be counted or measured:

1) Discrete units or events: cars, people, desks, jumps, bounces, games, bottles, cookies, etc.

2) Time: minutes, hours, seconds, days, weeks, years, etc.

3) Money: Dollars, Cents, other currencies

Quantities (or attributes) of objects that can be counted or measured:

4) Linear Units: inches, feet, centimeters, miles, light years, etc.

5) Area: square feet, square miles, square meters, etc.

6) Volume: cubic feet, cubic meters, cubic yards, gallons, liters, cups, teaspoons, etc.

Quantities (or attributes) of objects that can be counted or measured:

7) Weight: ounces, pounds, grams, tons, etc.

8) Energy: temperature, decibels, lumens, etc.

9) Angular Rotation degrees

Students should develop a fluency to discuss situations that compare two quantities through:

- > Scenarios (word problems and real life situations)
- > Tables
- > Equations
- > Graphs

Example: **Scenario:**

A store sells a package of 3 toy cars for \$5

Can you identify the **objects**,
quantities, and **units** that are
being counted and measured?

answer

Grade 6: Scenario: A store sells a package of 3 toy cars for \$5

Let's compare the number of cars to packages.

Number of Packages	calculations	Number of cars
1	given	3
2	$3+3$	6
3	$3+3+3$	9
4	$3(4)$	12
5	$3(5)$	15
10	$3(10)$	30
100	$3(100)$	300
p	$3(p)$	$c=3p$

Type A Questions: If I give you the number of packages, you tell me the number of cars

Type B Questions: If I give you the number of cars, you tell me the number of packages.

Grade 6: Scenario: A store sells a package of 3 toy cars for \$5

Let's compare the number of packages to the cost.

Number of Packages	calculations	Cost of packages
1	given	5
2	$5+5$	10
3	$5+5+5$	15
4	$5(4)$	20
5	$5(5)$	25
10	$5(10)$	50
100	$5(100)$	500
p	$5(p)$	$d=5p$

Type A Questions: If I give you the number of packages, you tell me the cost.

Type B Questions: If I give you the cost of the packages, you tell me the number.

As mathematics becomes more
involved
and the students get better with
progressions,
these tables can be combined

Grades 6 and 7:

Let p represent the **number** of packages.

Then c (equal to $3p$) represents the **number of toy cars**

and d (equal to $5p$) represents the **cost in dollars**.

Let's compare the number of cars to the cost of the packages.

Number of Packages (p)	Number of cars (c)	Cost of packages (d)
1	3	5
2	6	10
3	9	15
4	12	20
5	15	25
10	30	50
100	300	500
p	$c=3p$	$d=5p$

From here, we can ask and answer any questions about these quantities.

1. How many cars can I buy for \$20?
2. How much will it cost for 15 cars?
3. How many cars can I buy for \$30?
4. How much will it cost for 24 cars?
5. If sold separately, how much should one car cost?

Note:

Extensions: As the students become comfortable with progressions, they can reduce the number of steps in the process.

Number of cars	Cost of packages
3	5
6	10
9	15
12	20
15	25
30	50
300	500
c	$d = \frac{5}{3}c$

Transition to Grades 7,8, and Algebra

If a \$6 shipping fee is added to the scenario:

Number of cars	Cost of packages
0	6
3	11
6	16
9	21
12	26
15	31
30	56
300	506
$c=3p$	$d= \frac{5}{3} c+ 6$

equations

In the previous problem, we ended with:

$$d = \frac{5}{3}c + 6$$

If we switch the "problem-specific" variables to more generic x and y , we get the more familiar:

$$y = \frac{5}{3}x + 6$$

In 8th grade, we transition over to "function" notation. This would transition the above equation to:

$$f(x) = \frac{5}{3}x + 6$$

Later, students may be asked to compare different functions for the same values of x , so they may be asked to look at other functions which may be called $g(x)$, or $h(x)$. More complex mathematics (at high school and beyond) often uses or compares multiple functions.