**Teaching with Progressions**

What is a **Progression**? A progression looks like a modified IN/OUT (function) Table. The modifications are made to help structure the students’ thought process to move from concrete, to representational, to algebraic.

What **standards** will Progressions support? Progressions are great for linear expressions and proportional relationships, but they will support quadratic relationships as well as basic numeric relationships. (You may also notice a crossover of questions here in the GUES strategy, as they mesh nicely together.)

**Why** teach progressions? Progressions are a great way to relate concrete mathematics to algebra as well as create a bridge for students to access the information.

**Suggestion**: Begin by teaching students the following definition of mathematics:

***“Mathematics is the study of things that can be counted or measured and the relationships between them.”***

**The 9 Quantities:**

You can pick ANY object and discuss these 9 quantities (or attributes) of those objects. Some of them may be rather silly (you are not likely to measure the temperature of a desk or a book), but you could. For any object, there are only 9 quantities that can be counted or measured:

|  |  |  |
| --- | --- | --- |
| **Discrete objects** **or occurrences** (such as books, pencils, or jumps) | **Distance** (length, width, height, perimeter) | **Area** (Number of unit squares needed to cover an object) |
| **Volume** (Number of unit cubes needed to fill an object) | **Weight or mass** (amount of downward force) | **Energy** (typically temperature through grade 8, but could include others (decibels, lumens, newtons, horsepower, etc.) |
| **Time** (days, weeks, months, years, hours, minutes, seconds, etc.) | **Cost** (dollars or cents) | **Angle of rotation** (degrees) |

Other “attributes” are relationships between these quantities. For example speed is the ratio of distance to time.

**Progressions begin with a scenario.** *For example: A math textbook is 5 cm wide.*

Ask: *“What is being the object being counted or measured? What attribute is being measured? What units are being used?”*

Answer:

*The object used is a math textbook.*

Attributes*:*

*The width of the textbook is 5 cm.*

*The discrete number of textbooks is 1.*

**Next step:** Create a three column table.

Ideally, this would be a standard “T-chart” with the independent variable on the left and the dependent variable on the right. Reserve the center column for the work. **Note***that the column headings are the same as the attributes that were just identified:*

**Scenario: *A math textbook is 5 cm wide.***

|  |  |  |
| --- | --- | --- |
| **Number of textbooks1 (t)** | **work2&3** | **Total width of textbooks (w)** |
| 1 | 5  | 5 cm |
| 2 | 5 + 5 | 10 cm |
| 3 | 5 + 5 + 5 | 15 cm |
| 4 | 5 + 5 + 5 + 5 or 5(4) | 20 cm |
| 5 | 5(5) | 25 cm |
| 10 | 5(10) | 50 cm |
| 100 | 5(100) | 500 cm |
| t | 5(t) or 5t | 5t cm OR w = 5t |

1. I always use 1, 2, 3, 4, 5, 10, 100, and x (t in this case), unless there is a very good reason to use other numbers. Students MAY use additional rows to access more information.
2. There are two bridges in this column that are critical to students’ understanding of algebra. The first occurs in row 4 (some students may be ready for it before then, others may need additional rows. The language that I use in row 4 to help make the bridge is “how many TIMES will I need to add this number?” For that reason, I have included both the addition representation and the multiplication representation in row 4. Students can make this jump when they are comfortable.
3. The bridge to Algebra is made easier if you keep the row number in the parentheses, as you can see in row x.

**Where do we go from here?**

Have students generate their own questions based on this relationship:

**A-type questions**: I give you a value for the independent variable, you find the dependent variable:

Ex: *8 books would require how much space on the shelf?*

**B-type questions:**  I give you a value for the dependent variable, you find the independent variable:

*Ex: How many books can I fit on a bookshelf that is 55 cm wide?*

[note that this will review inverse operation skills, leading to the development of finding or using an inverse function as well as solving similar problems].

**Consider variations in difficulty level:**

**Easy**: The answer is IN the table (utilizing x between 1 and 5, 10, or 100).

**Medium:** The answer COULD BE in the table, with a reasonable extension (both of the examples above would fit into this category).

**Hard:** The student would need the equation to reasonably calculate the correct answer: ex: If I stacked 255 books, how tall would the stack be? Or A stack of books is 150 cm tall. How many books are in it?

**Next step:**  Consider linear equations (grades 6-8 + Algebra):

Try to create a progression table for the following:

**Scenario**: A taxi charges $2 + $0.30 per mile.

Ask: *“What is being the object being counted or measured? What attribute is being measured? What units are being used?”*

Answer:

*The object used is a taxi ride*

Attributes*:*

*Distance: the length of the ride (unknown)*

*Cost of the ride*

**Next step:** Create a three column table.

Ideally, this would be a standard “T-chart” with the independent variable on the left and the dependent variable on the right. Reserve the center column for the work. **Note***that the column headings are the same as the attributes that were just identified:*

**Scenario:  *A taxi charges $2 + $0.30 per mile.***

|  |  |  |
| --- | --- | --- |
| **distance (d) in miles** | **work** | **Cost of the ride in dollars** |
| 01 | $2 | $2 |
| 1 | 2 + 0.30 | 2.30 |
| 2 | 2 + 0.30+0.30 | 2.60 |
| 3 | 2 + 0.30+0.30 + 0.30 | 2.90 |
| 4 | 2 + 0.30+0.30 + 0.30+0.30 = 2 + 0.30(4) | 3.20 |
| 5 | 2 + 0.30(5) | 3.50 |
| 10 | 2 + 0.30(10) | 5.00 |
| 100 | 2 + 0.30(100) | 32.00 |
| t | 2 + 0.30(t) | 2 + .30t or C = 2 + .30t |

**A-type questions**: I give you a value for the independent variable, you find the dependent variable:

Easy: How much would a 4 mile ride cost?

Medium: How much would a 12 mile ride cost?

Hard: How much would an 87 mile ride cost?

**B-type questions:**  I give you a value for the dependent variable, you find the independent variable:

Easy: How far could I go on $2.90?

Medium: How far could you go on $6.10?

Hard: How far could you go on $20?

1: I like to keep row zero in when there is a starting value. Although it is unrealistic that you would actually pay $2 to sit in a taxi and not go anywhere, it helps the math. Otherwise, the graph is not actually linear if they go with $0 for 0 miles.

This can be used with any ratio, rate, or proportion problem, as well as nearly all linear equation word problems.