Teaching/Re-Teaching Basic Computation Facts

Adapted by Michael Braverman 10/29/2018

Adapted from Paul Riccomini: “Maximize all Students’ Mathematical Learning”

**Fluency requires:**

* 1. Specific criterion for introducing new facts
	2. Intensive practice on newly introduced facts (more than 1x)
	3. Systematic practice on previously introduced facts
	4. Adequate allotted time (5-10 min/day)
	5. Record keeping
	6. Motivational procedures
	7. Roughly 30 exposures per fact to move facts to long term memory and “instant recall”

**Practice, rehearsal and study of basic facts**, ***with fidelity***, will improve student performance on basic computational tasks, as well as make other operations, algorithms, and relationships easier for the students see, learn, understand, and answer competently.

**Procedure:**

1. Prior to initial assessment, you should have:

* An Initial Facts Inventory grid
* An answer grid
* A Study-Order grid
* A Mastered Fact Grid (for student self-monitoring)
* Two flash card “foldables”
* A zipper-style sandwich bag, rubber band, paper clip, etc. so your student can store grids and flash cards together.
* Scissors to use to separate cards

In addition, keep a “stack” of flash card “foldables” on hand, as you can anticipate your child to need 16 additional cards each session.

2. **Initial assessment:**

1. Give your child a blank facts grid ***face down***, and complete the first section of the analysis page [the back].
2. Tell them that they will be timed on the next activity. Suggest that they skip any problems they do not know immediately.
3. When all students are ready, have them turn the paper over and begin. Give them 6 minutes to complete the grid*. [169 blanks @ 2 seconds per blank = 338 seconds…6 minutes = 360 seconds. Ideally, the students will be able to complete the entire grid in* ***less than 180 seconds*** *once all facts have been mastered]*
4. After 6 minutes, have them put their pencils down. Then display or provide an “answer” grid.
5. Have them identify items they got wrong and items they did not get to (as well as items they spent a lot of time on.)
6. Provide a “practice” grid to your student. Have them place a check mark in the boxes that they know [mastered facts].
7. Then, starting with the upper left-hand corner of the page, and moving to the right and down systematically, one row at a time, have them identify the facts that they need to learn, in sets of four (complete section 2 on the back).
8. For each of the first four facts, they should create ***four*** identical flash cards (16 cards total). On the front of each card should be the problem (ex: 4 x 6 =). On the back of the card should be the entire fact family:

4 x 6 = 24, 6 x 4 = 24, 24 ÷ 4 = 6, 24 ÷ 6 = 4.

3. **Practice sessions:**

Students should practice (from their original “deck” of 16 flash cards) 5-10 minutes per day until they have reached ***automaticity*** on those facts, following the procedure outlined below.

* Ex: Front of card: 4 x 5 =

Back of card: 4 x 5 = 20

 5 x 4 = 20

 20 ÷ 4 = 5

 20 ÷ 5 = 4

* Students must say the fact (4 x 5 = 20)
	+ Question AND answer: “Four times five equals twenty”
* **Error Correction Procedure:**
	+ The only correct response is the correct answer to the fact\*.
	+ All other responses should be corrected
		- For example….Saying incorrect fact, hesitation, or calculation
	+ Stop student and say correct answer (I say)
	+ Say correct answer with student (We say)
	+ Have student say correct answer (You say)
	+ Student says correct answer (I say).
	+ Student reads the entire back of the card (fact family)
	+ Fact is placed three cards back to make sure student has opportunity to re-practice the fact while the correction is still in short term memory

\*The facts that students get correct (perfectly…on the first try for each time they appear) become “mastered.”

4. **After Practice session**:

Students should:

1. update their “mastered facts” grids
2. create flash cards for the next four facts on their list (see step “2g” above) and
3. remove two of each copy of their original facts (leaving their deck with 16 brand new cards, and 8 of the original 16 cards).
4. continue practice in this manner according to Riccomini Handout 6



Notes:
**The goal should be *fluency* and *automaticity***.

If you think about the number of basic operations that are required for more complicated computations, students should be able to do the basics with 100% competency and automaticity. For example, the problem:

$8\frac{1}{8}-2\frac{3}{5}=\\_\\_\\_$ would typically be solved with 8 basic operations as:

$8\frac{5}{40}-2\frac{24}{40}=$

$7\frac{45}{40}-2\frac{24}{40}=5\frac{21}{40}$

Which does not count the steps necessary to check the work.

**Mathematically speaking**, students should have the 0 and 1 times table immediately and flawlessly (or get them in fairly short order from the **Zero Property** And **Identity Property** of multiplication).

***Then***: there are 11 twos, 10 threes, 9 fours, etc. down to 1 twelve. This makes **60 unique facts** (one of the patterns to point out is the commutative property: 3 + 4 = 4 + 3. This is the reason students should write down the entire fact family on their flash cards.)

[Most students should have the 2, 5, and 10 times tables (or get them) in fairly short order from their inherent patterns.]

**Rehearsal**: By writing the fact families 4 times, the students end up “rehearsing” the fact 16 times BEFORE the first practice session, but they are focusing on only 4 facts, which will promote fact retention and numerical relationships. After the first cycle through the cards, they will have seen each fact at least 20 times (more if they made an error, had to count, or hesitated. Remember: 30 is the goal). Once the fact is learned, they will still review it twice per card cycle.

**Have the students track their progress and mastery**. Provide praise, incentives, or other positive feedback for mastering new facts as well as mastering the entire table. Feel free to give a new “initial” quiz after a week or two of regular practice sessions for students to see improvement.

**Show relevance:** Provide opportunities to see how often the basic facts are used in the regular lessons (multi-digit operations, fraction operations, etc.)

**Initial Facts Inventory**

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Mastered Fact Grid:

I know these!

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Section 1:

 Complete Name and Date above.

Section 2:

 Number correct: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ out of 169 = \_\_\_\_\_\_\_\_\_\_\_ %

* 1. First 4 facts to work on: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Date completed: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* 1. Next 4 facts to work on: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Date completed: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

* 1. Next 4 facts to work on: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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* 1. Next 4 facts to work on: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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**Multiplication Facts**

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| **2** | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| **3** | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| **4** | 0 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| **5** | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| **6** | 0 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| **7** | 0 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| **8** | 0 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| **9** | 0 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| **10** | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| **11** | 0 | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| **12** | 0 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

**Multiplication Facts – “study order”**

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| **1** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| **2** | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| **3** | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| **4** | 0 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| **5** | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| **6** | 0 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| **7** | 0 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| **8** | 0 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| **9** | 0 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| **10** | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| **11** | 0 | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| **12** | 0 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

**Addition Facts**

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| **1** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| **2** | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| **3** | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| **4** | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| **5** | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| **6** | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| **7** | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| **8** | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **9** | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| **10** | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| **11** | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| **12** | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |

**Addition Facts – “study order”**

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| **1** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| **2** | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| **3** | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| **4** | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| **5** | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| **6** | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| **7** | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| **8** | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **9** | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| **10** | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| **11** | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| **12** | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |

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